On 3d $\mathcal{N}{=}8$ Lorentzian BLG theory as a scaling limit of 3d superconformal $\mathcal{N}{=}6$ ABJM theory

E. Antonyan[*] and A.A. Tseytlin[†]

Theoretical Physics Group
The Blackett Laboratory, Imperial College,
London SW7 2AZ, U.K.

Abstract

We elaborate on the suggestion made in arXiv:0806.3498 that the 3d $\mathcal{N} = 8$ superconformal $SU(N)$ Chern-Simons-matter theory of "Lorentzian" Bagger-Lambert-Gustavson type (L-BLG) can be obtained by a scaling limit (involving sending the level k to infinity and redefining the fields) from the $\mathcal{N} = 6$ superconformal $U(N) \times U(N)$ Chern-Simons-matter theory of Aharony, Bergman, Jafferis and Maldacena (ABJM). We show that to implement such limit in a consistent way one is to extend the ABJM theory by an abelian "ghost" multiplet. The corresponding limit at the 3-algebra level also requires extending the non-antisymmetric Bagger-Lambert 3-algebra underlying the ABJM theory by a negative-norm generator. We draw analogy with similar scaling limits discussed previously for bosonic Chern-Simons theory and comment on some implications of this relation between the ABJM and L-BLG theories.

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[*]Also at the Institute for Theoretical Physics and Modeling, Yerevan, Armenia
[†]Also at the Lebedev Institute, Moscow.

1 Introduction

An important problem that attracted much recent attention is to understand a low-energy limit of a hypothetical theory that generalizes the worldvolume theory of a single M2-brane [1] to the case of N coincident M2-branes. Using some earlier ideas of [2], Bagger and Lambert, and Gustavson (BLG) succeeded in constructing a three-dimensional $\mathcal{N} = 8$ superconformal Chern-Simons-matter theory based on a 3-algebra [3, 4, 5]. However, it was soon realized that the original BLG theory can describe only two coincident M2-branes if the 3-algebra is kept finite and having a positive-definite metric [6, 7, 8].

An interesting suggestion of how to construct a similar theory with right symmetries and degrees of freedom to describe N M2-branes was made in [9, 10, 11] (see also [12, 13, 14, 15]) where a 3-algebra with a Lorentzian (indefinite) signature metric was used. The resulting Lorentzian-BLG (L-BLG) theory is formally $\mathcal{N} = 8$ superconformal at the classical level, but its interpretation as a quantum theory appears to be non-trivial (in particular, as there is no obvious expansion parameter and its perturbative definition depends on a choice of a vacuum). If expanded near an obvious classical vacuum that spontaneously breaks the superconformal symmetry, it becomes equivalent [10, 12, 14, 16] to the standard low-energy gauge theory of multiple D2-branes (non-conformal $\mathcal{N} = 8$ supersymmetric 3d SYM theory), i.e. it may thus be viewed as a conformal "dressing" of non-conformal 3d $\mathcal{N} = 8$ SYM theory.[1]

A different 3d superconformal Chern-Simons-matter theory was proposed by Aharony, Bergman, Jafferis and Maldacena (ABJM) [18]; it has an explicit $\mathcal{N} = 6$ supersymmetry [19] and may be interpreted as describing N coincident M2-branes at the singularity of the orbifold $\mathbb{C}^4/\mathbb{Z}_k$. For $N = 2$ it should be equivalent to the BLG theory in the $SU(2) \times SU(2)$ formulation [6].

While the ABJM theory also admits a 3-algebra interpretation [21] (where antisymmetry condition is relaxed, see also [22]) it appears to be very different from the L-BLG one: the two theories have different field content and different symmetries.

In ref.[23] an interesting suggestion was made that the L-BLG theory may be interpreted as a certain limit of the ABJM theory, in which one sends the ABJM coupling k (CS level) to infinity and at the same time rescales some of the fields to zero so that they decouple. However, it was not shown in [23] that such limit of the $\mathcal{N} = 6$ ABJM action does indeed lead to the full $\mathcal{N} = 8$ supersymmetric L-BLG action.

Here we shall refine the suggestion of [23] by pointing out that to be able to relate the two theories by a scaling limit one needs to supplement the ABJM theory with an extra abelian "ghost" multiplet (decoupled from the ABJM fields). We shall demonstrate that there exists a limit (or "contraction") of the 3-algebra underlying the ABJM theory [21] trivially extended by an extra "ghost" generator that leads exactly to the Lorentzian 3-algebra underlying the L-BLG theory [9, 10, 11]. The BLG construction of the superconformal theory from a 3-algebra then guarantees that the corresponding limit of the (extended) ABJM action is indeed the full

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[1]There is a close analogy with a conformal plane-wave 2d sigma model [17] having $D + 2$ dimensional Lorentzian target space $ds_{D+2}^2 = dx^+ dx^- + G_{ij}(x^+, x) dx^i dx^j$ which may be viewed as a "dressing" of non-conformal 2d sigma model with the Euclidean D dimensional target space metric $ds_D^2 = G_{ij}(x) dx^i dx^j$ that depends on x^+ according to the RG equation $\frac{\partial G_{ij}}{\partial x^+} = \beta_{ij}(G)$.

L-BLG action. This relation between 3-algebras reinforces the idea of [21] that 3-algebras may be an essential part of the multiple M2-brane theory.

The scaling limit we shall consider is very similar to the limits considered previously for 2d WZW models [24, 25], 3d Chern-Simons and 4d Yang-Mills theories [26] where one goes (via an infinite "boost" in field space and a rescaling of coupling) from a theory defined by a product of a "ghost" (time-like) direction and a simple Lie group to a theory defined by non-semisimple contraction of the corresponding algebra with non-degenerate (but indefinite) metric.[2]

In the 3d CS example considered in [26] the starting theory was the $U(1)_{-k} \times SU(2)_k$ Chern-Simons one and the limiting theory was the CS model based on the centrally extended Euclidean algebra E_2^c in 2 dimensions. Here we shall start with the $[U(N)_k \times U(N)_{-k}]$ Chern-Simons-matter $\mathcal{N} = 6$ ABJM theory [18, 21] with an extra decoupled "ghost" multiplet and end up with the $SU(N)$ Chern-Simons-matter $\mathcal{N} = 8$ L-BLG theory [9, 10, 11].

One motivation for considering this limit is that it may be possible to view it as a definition of the L-BLG theory in terms of the ABJM theory and that may shed light on the interpretation of the former. In particular, we shall see how the relation between the L-BLG theory and the 3d $\mathcal{N} = 8$ SYM (or D2-brane) theory can be understood in the ABJM framework: taking the scaling limit and then giving one of the scalars an expectation value is actually equivalent to the Higgs-type procedure of [16, 20] for obtaining the D2-brane theory from the ABJM theory.

We shall start in section 2 with defining the relevant 3-algebras and reviewing the ABJM and L-BLG theories. After reviewing in section 3.1 the scaling limit in the bosonic CS theory we shall then explain in sect 3.2 how to relate the kinetic terms in the ABJM action combined with a "ghost" multiplet to those of the L-BLG theory by a similar scaling limit. We will proceed to show in section 3.3 how the scaling limit transforms the ABJM 3-algebra "tensored" with a ghost generator into the Lorentzian BLG 3-algebra. We shall draw some conclusions in section 4, commenting in particular on the relation of the two theories to the D2-brane theory.

2   Review of ABJM and L-BLG theories

Let us start with a review of basic definition of 3-algebra that can be used [21] to construct the interaction terms in the supersymmetric CS-matter theories like ABJM and L-BLG.

A 3-algebra is a (complex) vector space with a basis T^a, $a = 1, \ldots, M$, endowed with a triple product

$$[T^a, T^b; T^c] = f^{abc}{}_d T^d , \tag{2.1}$$

where the structure constants satisfy the following fundamental identity

$$f^{efg}{}_b f^{cba}{}_d + f^{fea}{}_b f^{cbg}{}_d + f^{*gaf}{}_b f^{ceb}{}_d + f^{*age}{}_b f^{cfb}{}_d = 0 . \tag{2.2}$$

Here we assume a general form of a 3-algebra, for which structure constants are a priori anti-symmetric only in the first 2 upper indices [21, 27].

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[2]In 2 dimensions this limit is a special case of a "Penrose-type" limit that leads to a plane-wave-type sigma model [25]. The infinite rescaling of the overall coefficient of the action (or string tension) which is part of the limiting procedure implies that the resulting model is conformally invariant. Let us note also that the enhancement of supersymmetry in Penrose limit is a well-known phenomenon.

It was shown in [4, 5] that when $f^{abc}{}_d$ are real and antisymmetric in a, b, c, one can construct a set of 3d equations of motion that are invariant under 16 supersymmetries and $SO(8)$ R-symmetry. Adding an assumption of existence of an invariant inner product

$$h^{ab} = \langle T^a, T^b \rangle \,, \tag{2.3}$$

allows one to construct the Chern-Simons-matter $\mathcal{N} = 8$ superconformal BLG action [5]. This applies, in particular, for the $SU(N)$ Lorentzian BLG theory of [9, 10, 11].

This construction was further generalized in [21] to the case when the structure constants which are no longer real and antisymmetric in all three indices but are required to satisfy (in addition to (2.2)) the following condition

$$f^{abcd} = -f^{bacd} = -f^{abdc} = f^{*cdab} \,, \qquad f^{abcd} \equiv f^{abc}{}_e h^{de} \,. \tag{2.4}$$

It was shown in [21] that such more general algebras lead, in particular, to the $\mathcal{N} = 6$ Chern-Simons-matter theory of ABJM [18].

Let us now review the Lagrangians of the two theories we are interested in. The ABJM theory [18, 19] is invariant under 24 supercharge generators ($\mathcal{N} = 6$ superconformal symmetry), $SU(4)$ R-symmetry, and a $U(1)$ internal symmetry. The field content is given by the $U(N)_k \times U(N)_{-k}$ CS gauge field $(A_\mu^{(L)}, A_\mu^{(R)})$ and bi-fundamental ($N \times N$ matrix-valued) matter fields – 4 complex scalar fields Y^A ($A = 1, 2, 3, 4$), and their hermitian conjugates $Y_A^\dagger$, as well as the fermions ψ_A and their hermitian conjugates $\psi^{A\dagger}$. Fields with raised A index transform in the **4** of the R-symmetry $SU(4)$ group and those with lowered index transform in the **4̄**. The corresponding Lagrangian has the following form

$$
\begin{aligned}
\mathcal{L}_{\text{ABJM}} &= \mathcal{L}_{\text{CS}}(A) - \text{tr}(D_\mu Y^A D^\mu Y_A^\dagger) - V(Y) - i\text{tr}(\bar{\psi}^{A\dagger} \gamma^\mu D_\mu \psi_A) \\
&\quad - i\frac{2\pi}{k}\text{tr}(\bar{\psi}^{A\dagger}\psi_A Y_B^\dagger Y^B - \bar{\psi}^{A\dagger}Y^B Y_B^\dagger \psi_A) + 2i\frac{2\pi}{k}\text{tr}(\bar{\psi}^{A\dagger}\psi_B Y_A^\dagger Y^B - \bar{\psi}^{A\dagger}Y^B Y_A^\dagger \psi_B) \\
&\quad + i\frac{2\pi}{k}\varepsilon_{ABCD}\text{tr}(\bar{\psi}^{A\dagger}Y^C \psi^{B\dagger}Y^D) - i\frac{2\pi}{k}\varepsilon^{ABCD}\text{tr}(Y_D^\dagger \bar{\psi}_A Y_C^\dagger \psi_B) \,.
\end{aligned} \tag{2.5}
$$

Here $\mathcal{L}_{\text{CS}}$ is a Chern-Simons term and $V(Y)$ is a sextic scalar potential

$$
\mathcal{L}_{\text{CS}} = \frac{k}{4\pi}\epsilon^{\mu\nu\lambda}\text{tr}\left[A_\mu^{(L)}\partial_\nu A_\lambda^{(L)} + \frac{2i}{3}A_\mu^{(L)}A_\nu^{(L)}A_\lambda^{(L)} - A_\mu^{(R)}\partial_\nu A_\lambda^{(R)} - \frac{2i}{3}A_\mu^{(R)}A_\nu^{(R)}A_\lambda^{(R)}\right] \,, \tag{2.6}
$$

$$
\begin{aligned}
V(Y) = -\frac{4\pi^2}{3k^2}\text{tr}\Big(&Y^A Y_A^\dagger Y^B Y_B^\dagger Y^C Y_C^\dagger + Y_A^\dagger Y^A Y_B^\dagger Y^B Y_C^\dagger Y^C \\
&+ 4Y^A Y_B^\dagger Y^C Y_A^\dagger Y^B Y_C^\dagger - 6Y^A Y_B^\dagger Y^B Y_A^\dagger Y^C Y_C^\dagger\Big) \,.
\end{aligned} \tag{2.7}
$$

This action is expected to provide a low-energy description of N M2-branes at an $\mathbb{C}^4/\mathbb{Z}_k$ orbifold singularity [18].

In [21] the general form for the action of a 3d scale-invariant field theory with $\mathcal{N} = 6$ supersymmetry, $SU(4)$ R-symmetry and $U(1)$ global symmetry was found by starting with a 3-algebra in which the triple product is not antisymmetric. The field content is the same as

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described above (with the matter fields taking values in the 3-algebra, i.e. $Y^A = T^a Y_a^A$, $\psi_A = T^a \psi_{Aa}$, while the gauge field is $A_\mu = A_{\mu ab} T^a \otimes T^b$) and the corresponding Lagrangian may be written as

$$
\begin{aligned}
\mathcal{L} = {} & \mathcal{L}_{\text{CS}} - \langle D^\mu Y_A, D_\mu Y^A \rangle - V - i \langle \bar{\psi}^A, \gamma^\mu D_\mu \psi_A \rangle \\
& - i \langle \bar{\psi}^A, [\psi_A, Y^B; Y_B] \rangle + 2i \langle \bar{\psi}^A, [\psi_B, Y^B; Y_A] \rangle \\
& + \frac{i}{2} \varepsilon_{ABCD} \langle \bar{\psi}^A, [Y^C, Y^D; \psi^B] \rangle - \frac{i}{2} \varepsilon^{ABCD} \langle Y_D, [\bar{\psi}_A, \psi_B; Y_C] \rangle ,
\end{aligned} \tag{2.8}
$$

where the brackets stand for the inner product in 3-algebra (2.3), $D_\mu Y_a^A = \partial Y_a^A - A_{\mu bd} f^{cdb}{}_a Y_c^A$ and

$$
\mathcal{L}_{\text{CS}} = \frac{1}{2} \epsilon^{\mu\nu\lambda} \left(f^{abcd} A_{\mu cb} \partial_\nu A_{\lambda da} + \frac{2}{3} f^{acd}{}_g f^{gefb} A_{\mu ab} A_{\nu dc} A_{\lambda fe} \right) , \tag{2.9}
$$

$$
V = \frac{2}{3} \langle \Upsilon_B^{CD}, \Upsilon_{CD}^B \rangle , \quad \Upsilon_B^{CD} = [Y^C, Y^D; Y_B] - \frac{1}{2} \delta_B^C [Y^E, Y^D; Y_E] + \frac{1}{2} \delta_B^D [Y^E, Y^C; Y_E]
$$

Choosing a particular matrix realization of the 3-algebra such that for any 3 elements X_1, X_2, X_3 one has [21]

$$
[X_1, X_2; X_3] = \kappa (X_1 X_3^\dagger X_2 - X_2 X_3^\dagger X_1) , \qquad \kappa \equiv \frac{2\pi}{k} , \tag{2.10}
$$

and an inner product given by an ordinary matrix trace

$$
\langle X_1, X_2 \rangle = \text{tr}(X_1^\dagger X_2) , \tag{2.11}
$$

one observes that (2.8) becomes the same as the ABJM Lagrangian (2.5). The advantage of this form of the action (2.8) is that its structure is completely determined by the underlying 3-algebra.

In the particular case of the totally antisymmetric 3-algebras the corresponding action [5] has the enhanced $\mathcal{N} = 8$ supersymmetry. While there is only one non-trivial antisymmetric 3-algebra with a Euclidean metric [7], choosing a Lorentzian metric one finds an infinite class of 3-algebras with an underlying Lie algebra structure [8]. For any Lie algebra $\mathcal{G}$

$$
[T^i, T^j] = f^{ij}{}_k T^k \tag{2.12}
$$

with structure constants $f^{ij}{}_k$ and Killing form h^{ij} one can define the corresponding 3-algebra as follows. Let the generators T^a of the 3-algebra be denoted by T^-, T^+, T^i ($a = +, -, i$; $i = 1, \ldots, \dim \mathcal{G}$), where T^i are in one-to-one correspondence with the generators of the Lie algebra. Then the basic 3-algebra relations are chosen to be

$$
[T^-, T^a; T^b] = 0 , \qquad [T^+, T^i; T^j] = f^{ij}{}_k T^k , \qquad [T^i, T^j; T^k] = -f^{ijk} T^- , \tag{2.13}
$$

where a, b take $(+, -, i)$ values and $f^{ijk} \equiv f^{ij}{}_l h^{lk}$ is totally antisymmetric, i.e. this 3-algebra is antisymmetric. The invariant inner product is defined as follows

$$
\begin{aligned}
\langle T^-, T^- \rangle = 0 , \quad & \langle T^-, T^+ \rangle = 1 , \quad & \langle T^-, T^i \rangle = 0 , \\
\langle T^+, T^+ \rangle = b , \quad & \langle T^+, T^i \rangle = 0 , \quad & \langle T^i, T^j \rangle = h^{ij} .
\end{aligned} \tag{2.14}
$$

5

Here b is an arbitrary constant; since redefining $T^+ \to T^+ + aT^-$ preserves the 3-algebra structure but shifts $b = \langle T^+, T^+ \rangle \to b - 2a$, we can always choose $b = 0$.

Using this Lorentzian 3-algebra in the $\mathcal{N} = 8$ supersymmetric BLG construction (now with real scalar fields X^I ($I = 1, ..., 8$), fermions and gauge fields carrying 3-algebra indices $+, -$ or i, where we may choose $i = 1, .., N^2 - 1$ to be the index the adjoint representation of the $SU(N)$ Lie algebra) one finds the following Lagrangian [9, 10, 11][3]

$$
\begin{aligned}
\mathcal{L}_{\mathrm{L-BLG}} \;=\; \mathrm{tr}\Big[& -\frac{1}{2}\big[D_\mu(A)X^I - B_\mu X_+^I\big]^2 + \frac{1}{4}(X_+^K)^2([X^I, X^J])^2 - \frac{1}{2}(X_+^I[X^I, X^J])^2 \\
& + \frac{i}{2}\bar{\Psi}\Gamma^\mu D_\mu(A)\Psi + i\bar{\Psi}_+\Gamma^\mu B_\mu \Psi - \frac{1}{2}\bar{\Psi}_+ X^I[X^J, \Gamma_{IJ}\Psi] + \frac{1}{2}\bar{\Psi}X_+^I[X^J, \Gamma_{IJ}\Psi] \\
& + \frac{1}{2}\epsilon^{\mu\nu\lambda}F_{\mu\nu}B_\lambda - \partial^\mu X_+^I\,B_\mu X^I\Big] + \mathrm{tr}\Big[-\partial^\mu X_+^I\partial_\mu X_-^I + \frac{i}{2}\bar{\Psi}_-\Gamma^\mu \partial_\mu \Psi_+\Big]. \quad (2.15)
\end{aligned}
$$

This theory having the right symmetries and an arbitrary-rank $SU(N)$ gauge group was proposed [9, 10] as a candidate for a low-energy limit of a theory on N M2-branes in flat space.[4]

Like similar models in two [24] and higher [26] dimensions it has a scale symmetry that allows one to absorb the coefficient in front of the Lagrangian into a field redefinition. It thus has no obvious coupling constant and thus no natural perturbative expansion preserving all the symmetries (if one chooses a particular expansion point in field space one can use the standard loop expansion but a non-zero background would spontaneously break the conformal invariance).

Assuming one is allowed to consider only the observables that do not depend on X_-, one may integrate over X_- getting a delta-function constraint $\partial^2 X_+ = 0$. Solving it by $X_+ = v$=const and integrating out B_μ one obtains [9]–[14], via simple path integral duality transformation [16], the 3d $SU(N)$ $\mathcal{N} = 8$ SYM theory with the coupling constant $g_{YM} = v$, i.e. one establishes a relation between the L-BLG theory and the low-energy theory of N coincident D2-branes.[5]

In principle, this does not necessarily need to imply that the L-BLG theory (2.15) is equivalent to D2-brane theory – this is so only if it is treated in perturbative expansion near the vacuum $X_+ = v$=const which spontaneously breaks conformal symmetry and introduces v as a coupling constant. To preserve the conformal symmetry one may insist on summing over all solutions of $\partial^2 X_+ = 0$, and, in particular, on integrating over v (cf. [13, 15, 23]). It remains to be seen if one can give a precise meaning to such "summation" [15] or "integration over coupling constant" (which obviously depends on a definition of a measure of integration, etc.).

Our aim below will be to make precise, following the suggestion of [23], in which sense the L-BLG theory (2.15) can be interpreted as a scaling limit of the ABJM theory (2.5). One may

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[3]Here we set $A_\mu^i = A_{\mu i+}$, $B_\mu^i = \frac{1}{2}f^{ijk}A_{\mu jk}$ and used that $A_{\mu+-}$ and $A_{\mu-i}$ decouple. We then replaced the fields with adjoint index i with matrices in the fundamental representation of $SU(N)$ (i.e. X_i^I, etc, by $N \times N$ hermitian matrices) and thus replaced summation over i (with $h_{ij} = \delta_{ij}$) by the trace.

[4]In contrast to similar plane-wave sigma models where the existence of a negative norm direction in field space is natural and does not represent a problem due to reparametrization invariance allowing to fix a light-cone gauge, here there is an apparent non-unitarity issue. It is possible that it can be avoided by a consistent truncation of the spectrum or by gauging the shifts in X_- direction [12, 13].

[5]In a sense, the L-BLG theory can be interpreted as a "conformal dressing" of the 3d $\mathcal{N} = 8$ SYM theory where one effectively integrates over its coupling constant. This does not, however, mean that the two theories are completely equivalent, cf. footnote 1 and a discussion below.

hope that understanding the L-BLG theory via this scaling limit may possibly shed light on its proper interpretation and suggest an alternative way of defining the quantum L-BLG theory without explicit breaking of its conformal invariance. As we shall see, to implement such a scaling limit in a systematic fashion one will need to supplement the ABJM theory by an extra "ghost" supermultiplet, and thus it may not help with clarifying the unitarity issue of L-BLG theory.

3 Scaling limit

Here we shall first review the scaling limit of Chern-Simons theory and then show how a similar limit can be used to relate the ABJM action to L-BLG action by first adding an extra singlet ghost multiplet to the ABJM theory.

Next, we shall show that there exists a scaling limit or contraction of the non-antisymmetric 3-algebra (2.10) associated to the $U(N) \times U(N)$ gauge group [21] supplemented by an extra negative-norm generator that reduces it to the antisymmetric Lorentzian 3-algebra associated to the $SU(N)$ gauge group. This will provide a rigorous confirmation of the relation between the full non-linear ABJM and L-BLG actions via the scaling limit.

3.1 Examples of scaling limits of Chern-Simons theory

Let us start with recalling a simple example of a relevant type of scaling limit – the one that leads from the $U(1) \times SU(2)$ algebra to the centrally extended Euclidean algebra in 2 dimensions E_2^c. The idea [25] will be to mix together the $U(1)$ generator J^0 with a Cartan $U(1)$ generator J^3 of $SU(2)$. Denoting the remaining two generators of $SU(2)$ by J^n, $n = 1, 2$ we have

$$[J^0, J^n] = 0, \quad [J^0, J^3] = 0, \quad [J^n, J^m] = \epsilon^{nm} J^3, \quad [J^3, J^n] = \epsilon^{nm} J^m . \tag{3.1}$$

Let us introduce $\mathrm{J}^+, \mathrm{J}^-, \mathrm{J}^n$ by setting

$$J^0 = \varepsilon^{-2} \mathrm{J}^-, \qquad J^3 = \mathrm{J}^+ + \varepsilon^{-2} \mathrm{J}^-, \qquad J^n = \varepsilon^{-1} \mathrm{J}^n , \tag{3.2}$$

and take the limit $\varepsilon \to 0$. Then we end up with the algebra of E_2^c:[6]

$$[\mathrm{J}^-, \mathrm{J}^n] = 0, \qquad [\mathrm{J}^-, \mathrm{J}^+] = 0, \qquad [\mathrm{J}^+, \mathrm{J}^n] = \epsilon^{nm} \mathrm{J}^m , \qquad [\mathrm{J}^n, \mathrm{J}^m] = \epsilon^{nm} \mathrm{J}^- . \tag{3.3}$$

If we consider the corresponding quadratic form (or Casimir) on the original algebra taking the $U(1)$ part with negative sign we get $-J^0 J^0 + J^3 J^3 + J^n J^n = \varepsilon^{-2}(2\mathrm{J}^-\mathrm{J}^+ + \mathrm{J}^n\mathrm{J}^n)$ which is non-degenerate after an overall rescaling. While the standard Killing form on E_2^c is degenerate, it admits a non-degenerate invariant bilinear form with the signature $(-1, 1, 1, 1)$ with entries [24, 28]

$$\langle \mathrm{J}^m, \mathrm{J}^n \rangle = \delta_{mn}, \qquad \langle \mathrm{J}^m, \mathrm{J}^\pm \rangle = 0, \qquad \langle \mathrm{J}^-, \mathrm{J}^- \rangle = 0, \quad \langle \mathrm{J}^-, \mathrm{J}^+ \rangle = 1, \quad \langle \mathrm{J}^+, \mathrm{J}^+ \rangle = b, \tag{3.4}$$

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[6]An equivalent (up to $J_0 \to -J_0$ and shift of generators) prescription leading to the same algebra is to define $J^0 = \frac{1}{2}\mathrm{J}^+ - \varepsilon^{-2}\mathrm{J}^-$, $J^3 = \frac{1}{2}\mathrm{J}^+ + \varepsilon^{-2}\mathrm{J}^-$, $J^n = \varepsilon^{-1}\mathrm{J}^n$, i.e. $\mathrm{J}^+ = J^3 + J^0$, $\mathrm{J}^- = \frac{1}{2}\varepsilon^2(J^3 - J^0)$. Then $[\mathrm{J}^-, \mathrm{J}^n] = 0$ and we get back the rest of (3.3).

where b can be set to 0 by a redefinition of generators. This may be compared to (2.13),(2.14) where T^- corresponds to the central element J^- and T^+ – to the redefined rotation generator J^+.

As discussed in [26], the CS theory for E_2^c with the above non-degenerate invariant form can be obtained by a scaling limit from CS theory for $U(1)_{-k} \times SU(2)_k$: one should do a redefinition of gauge field components and take $k \to \infty$. Explicitly, starting with

$$S_{U(1)_{-k} \times SU(2)_k} = \frac{ik}{8\pi} \int d^3x \; \epsilon^{\mu\nu\lambda} \left(A_{\mu 0} \partial_\nu A_{\lambda 0} - A_{\mu 3} \partial_\nu A_{\lambda 3} - A_{\mu n} \partial_\nu A_{\lambda n} - \epsilon_{nm} A_{\mu n} A_{\nu m} A_{\lambda 3} \right), \quad (3.5)$$

and setting

$$A_{\mu 0} = -A_{\mu +} + \varepsilon^2 A_{\mu -} \; , \qquad A_{\mu 3} = A_{\mu +} \; , \qquad A_{\mu n} = \varepsilon A_{\mu n} \; , \qquad k = \varepsilon^{-2} \tilde{k} \; , \qquad (3.6)$$

and then taking the limit $\varepsilon \to 0$ (with A_a and $\tilde{k}$ fixed) we end up with

$$S_{E_2^c} = -\frac{i\tilde{k}}{8\pi} \int d^3x \; \epsilon^{\mu\nu\lambda} \left(2A_{\mu -} \partial_\nu A_{\lambda +} + A_{\mu n} \partial_\nu A_{\lambda n} + \epsilon_{nm} A_{\mu n} A_{\nu m} A_{\lambda +} \right) \; , \qquad (3.7)$$

which is the CS action for E_2^c with the metric (3.4) with $b = 0$.[7]

The redefinition in (3.6) is consistent with the one (3.2) used in taking the scaling limit in the algebra, namely,

$$A_0 J^0 + A_3 J^3 + A_n J^n = A_+ J^+ + A_- J^- + A_n J^n \; . \qquad (3.8)$$

Note that the action (3.7) does not actually depend on $\tilde{k}$ as it can be redefined away by rescaling the fields. This feature is shared by the L-BLG action – the overall coefficient in (2.15) can be set equal to 1 by a field redefinition [13].[8]

Let us mention also another example of a limit of CS theory that leads to a BF theory (cf. [29]). In this case we start with CS theory for the group $G_{-k} \times G_k$, e.g., $G = U(N)$. Let us denote the two gauge fields as $A_\mu^{(L,R)}$ and define their combinations A_μ and B_μ as

$$A_\mu^{(L)} = A_\mu - \tfrac{1}{2} \varepsilon B_\mu \; , \qquad A_\mu^{(R)} = A_\mu + \tfrac{1}{2} \varepsilon B_\mu \; , \qquad k = \varepsilon^{-1} \tilde{k} \; . \qquad (3.9)$$

Then taking the limit $\varepsilon \to 0$ (i.e. $k \to \infty$ for fixed $\tilde{k}$) in the corresponding action we end up with (cf. (3.7))

$$S = -\frac{i\tilde{k}}{8\pi} \int d^3x \; \epsilon^{\mu\nu\lambda} B_\mu F_{\nu\lambda}(A) \; . \qquad (3.10)$$

Here $\tilde{k}$ can be set to 1 by a redefinition of B_μ.

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[7]In general, the CS Lagrangian written in terms of an invariant metric Ω_{ab} and structure constants of the algebra $f^a{}_{bc}$ is $L = \Omega_{ad} \epsilon^{\mu\nu\lambda} \left(A_\mu^a \partial_\nu A_\lambda^d + \tfrac{1}{3} f^a{}_{bc} A_\mu^d A_\nu^b A_\lambda^c \right)$.

[8]One is to rescale the fields in (2.15) as follows: $X_i^I \to \kappa X_i^I$, $X_+^I \to \kappa^{-1} X_+^I$, $X_-^I \to \kappa^3 X_-^I$, $B \to \kappa^2 B$, $\kappa = k^{-1/2}$.

The corresponding limit at the level of gauge algebra can be defined as follows: if the generators of $G \times G$ are $T_i^{(L)}, T_i^{(R)}$ with both sets satisfying $[T_i, T_j] = f_{ij}{}^k T_k$, then we may define

$$T_i = T_i^{(R)} + T_i^{(L)} , \qquad P_i = \tfrac{1}{2}\varepsilon(T_i^{(R)} - T_i^{(L)}) . \tag{3.11}$$

In the limit $\varepsilon \to 0$ we will then get a semidirect sum of the algebra of G and translations, i.e.

$$[T_i, T_j] = f_{ij}{}^k T_k , \qquad [P_i, T_j] = f_{ij}{}^k P_k , \qquad [P_i, P_j] = 0 . \tag{3.12}$$

This relation of the BF structure of the CS part of the L-BLG action (2.15) to such Inonu-Wigner contraction-type scaling limit was mentioned earlier in [9, 10, 23].

3.2 From ABJM + "ghost" action to L-BLG action via a scaling limit

We are now ready to discuss the scaling limit of the ABJM action suggested in [23]. We shall concentrate on the bosonic part of the action (the fermions are readily included after we describe the corresponding scaling limit at the level of the 3-algebras below). In addition to the $U(N)_{-k} \times U(N)_k$ CS action the bosonic part of the ABJM action (2.5) contains also the "matter" part – complex $N \times N$ scalar field matrix Y^A ($A = 1, 2, 3, 4$) in bi-fundamental representation of $U(N) \times U(N)$ with the kinetic term $-\text{tr}(D_\mu Y^A D^\mu \bar{Y}_A)$, where $\bar{Y}_A = (Y^A)^\dagger$. Here

$$D_\mu Y^A = \partial_\mu Y^A + A_\mu^{(L)} Y^A - Y^A A_\mu^{(R)} = \partial_\mu Y^A + [A_\mu, Y^A] - \tfrac{1}{2}\varepsilon\{B_\mu, Y^A\} , \tag{3.13}$$

where we used the same field redefinition as in (3.9). If we now separate the trace part in Y^A by setting

$$Y^A = Y_0^A \mathbb{I} + \tilde{Y}^A, \qquad Y_0^A = \varepsilon^{-1} Y_+^A , \qquad \text{tr } \tilde{Y}^A = 0 , \tag{3.14}$$

we get 4 complex singlet fields Y_+^A and $4(N^2 - 1)$ complex scalar components of $\tilde{Y}^A$. We have introduce the factor of ε^{-1} in the singlet part to define the scaling limit leading to the L-BLG action. Then we get

$$-\text{tr}|D_\mu Y^A|^2 = -\text{tr}|\tilde{D}_\mu \tilde{Y}^A|^2 + \text{tr}|\varepsilon^{-1}\partial_\mu Y_+^A - \tfrac{1}{2}\varepsilon\{B_\mu, \tilde{Y}^A\}|^2 , \tag{3.15}$$

$$\tilde{D}_\mu \tilde{Y}^A \equiv \partial_\mu \tilde{Y}^A + [A_\mu, \tilde{Y}^A] - B_\mu Y_+^A . \tag{3.16}$$

Taking the limit $\varepsilon \to 0$ the second term in (3.15) gives

$$-\text{tr}|\varepsilon^{-1}\partial_\mu Y_+^A - \tfrac{1}{2}\varepsilon\{B_\mu, \tilde{Y}^A\}|^2 \to -N\varepsilon^{-2}|\partial_\mu Y_+^A|^2 + 2\big[\partial_\mu Y_+^A \text{tr}(B_\mu \tilde{Y}^A) + c.c.\big] . \tag{3.17}$$

As was observed in [23], the first term in (3.15) and the second term in (3.17) are as in the L-BLG action (2.15) with the 4 complex singlet scalars Y_+^A corresponding to the 8 real scalar fields X_+^I; the same was found to be true also for the bosonic interaction terms [23].[9]

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[9]The $U(1)$ component of the $U(N)$ field A_μ field automatically decouples, while the $U(1)$ component of the $U(N)$ field B_μ field can be decoupled by rescaling it by an extra factor of ε.

One is still left with a singular first term in (3.17), $\sim \varepsilon^{-2}|\partial_\mu Y_+^A|^2$. In [23] it was suggested that to make the action finite one is to require that $\partial^2 Y_+^A = 0$ which was then concluded to be the same as the equation in L-BLG theory (2.15) obtained by variation over X_-^I.

This suggestion, however, appears to be hardly satisfactory for several reasons:

(i) The condition for the vanishing of that singular term is, in general, ambiguous (depending, e.g., on boundary conditions) and, in fact, appears to require that $\partial_\mu Y_+^A = 0$. Then Y_+^A=const but in this case the L-BLG action becomes equivalent to 3d SYM or D2-brane action. The relation to the ABJM theory via a scaling limit with $k \to \infty$ is then not too surprising as the D2-brane theory is also a limit of the ABJM theory [18, 30]. The limit considered in [23] then does not represent a consistent derivation of the L-BLG theory from the ABJM one but rather of a D2-brane theory from the ABJM theory.

(ii) The non-trivial difference of the L-BLG theory from D2-brane theory is in the presence of extra 8 scalar fields X_-^I on which "observables" (composite conformal operators representing states) may, in general, depend and which may enter the external sources (cf. [15]). However, there is no place for such fields in the above limit of the ABJM theory considered in [23]: the matter fields Y^A or $Y_+^A, \tilde{Y}^A$ correspond only to X_+^I, X^I.

(iii) The above limit missing X_-^I fields is also not consistent with an expectation that the scaling limit may be may be carried out directly at the level of the corresponding 3-algebras: the Lorentzian 3-algebra contains the generator T^- corresponding to X_- (in addition to the generators corresponding to the $SU(N)$ algebra and the X_+ fields) but the 3-algebra for the ABJM theory lacks the corresponding generator. A related point is that while the scalar product on the Lorentzian 3-algebra vector space is indefinite, the scalar product on the ABJM 3-algebra vector space is hermitian (reflecting the positivity of the scalar kinetic term in the ABJM action).

Here we are going to improve the suggestion of [23] by proposing that in order to obtain the L-BLG theory by a consistent scaling limit similar to the one that relates the corresponding CS parts of the actions one is to extend the original ABJM theory by an extra "ghost" $U(1)$ multiplet containing 4 complex bosonic fields U^A and the corresponding singlet fermions. To implement such scaling limit at the 3-algebra level one will then need to extend the ABJM 3-algebra by an extra negative-norm generator.

Namely, we shall start with the (bosonic) ABJM Lagrangian supplemented by an extra term $N|\partial_\mu U^A|^2$ having "wrong" sign of its kinetic term. Then setting

$$U^A = -\varepsilon^{-1} Y_+^A + \varepsilon N^{-1} Y_-^A \, , \tag{3.18}$$

where Y_+^A is defined in (3.14) and Y_-^A is a new variable, and taking the limit $\varepsilon \to 0$ we shall then get instead of (3.17)

$$N|\partial_\mu U^A|^2 - \mathrm{tr}|\varepsilon^{-1}\partial_\mu Y_+^A - \tfrac{1}{2}\varepsilon\{B_\mu, \tilde{Y}^A\}|^2 \ \to \ -\partial_\mu Y_+^A \partial^\mu Y^A - +2\big[\partial^\mu Y_+^A \mathrm{tr}(B_\mu \tilde{Y}^A) + c.c.\big] \tag{3.19}$$

As a result, we recover the full kinetic term of the L-BLG theory (2.15) (with $X_\pm^I$ being the real parts of $Y_\pm^A$).

This limiting procedure is obviously similar to the one discussed above on the example of the $U(1)_{-k} \times SU(2)_k$ CS model or the one used in the string sigma model context in getting

a pp-wave model via a Penrose-type limit [25] (with U^A playing the role of the "target space time" direction).

With a hindsight, the need to extend the ABJM action by an extra "ghost" U^A field is hardly unexpected: it would be strange to obtain the L-BLG action which has an indefinite kinetic-term signature from a manifestly definite ABJM action by a regular scaling limit. The existence of this scaling limit does not seem to shed extra light on the unitarity issue of the L-BLG theory:[10] while at the level of the ABJM theory one may assume that the extra ghost multiplet is completely decoupled (and does not enter the observables), it gets effectively coupled via the redefinition (3.18) in the process of taking the scaling limit.

3.3 From ABJM + "ghost" 3-algebra to L-BLG 3-algebra via a scaling limit

Let us now show that starting with the 3-algebra (2.10) for the ABJM theory extended by an extra "ghost" generator one can get the Lorentzian 3-algebra (2.13) associated to L-BLG theory by a scaling limit. Since the two supersymmetric actions can be directly constructed from the corresponding 3-algebras [9, 10, 21], that will imply, in particular, that the scaling limit will go through at the level of the full actions including the fermions and all interaction terms.

The Lorentzian 3-algebra vector space has an indefinite scalar product (2.14) while that of the ABJM algebra (2.11) is positive definite. Counting the degrees of freedom in the $U(N) \times U(N)$ ABJM theory we get $4 \times 2N^2 = 8N^2$ scalars, while in the $SU(N)$ L-BLG theory we get $8 \times (1 + 1 + (N^2 - 1)) = 8N^2 + 8$ scalars. To match the degrees of freedom we thus need to add an extra generator to the ABJM 3-algebra; it should have a negative norm and should thus correspond to the "ghost" multiplet introduced in the previous subsection.

Let us start by considering the scaling limit for the 3-algebra corresponding to the simplest case of the $U(2) \times U(2)$ gauge group. We decompose the 2×2 complex matrices X in (2.10) (which may be identified with maps from $\mathbb{C}^2$ to $\mathbb{C}^{*2}$, or elements of bi-fundamental representation of $U(2) \times U(2)$ [21]) in the following basis (with complex coefficients): $\{E, \sigma^i\}$. Here $E = iI_2$ with I_2 as the 2×2 identity matrix and σ^i are the Pauli matrices. The 3-algebra relations (2.10) for these basic elements then become

$$[E, \sigma^i; \sigma^j] = [\sigma^i, \sigma^j; E] = 2\kappa \epsilon^{ijk} \sigma^k ,$$

$$[\sigma^i, \sigma^j; \sigma^k] = -2\kappa \epsilon^{ijk} E , \qquad \kappa = \frac{2\pi}{k} . \qquad (3.20)$$

Note that this $U(2) \times U(2)$ (or $N = 2$) case is special in that the 3-algebra is actually fully antisymmetric. This implies that in this case the corresponding action will have the extended $\mathcal{N} = 8$ supersymmetry [18]. [11]

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[10]Also, it does not seem possible to obtain the gauged version [12, 13] of the L-BLG theory by a scaling limit of (a version of) the ABJM theory.

[11]The reality condition implies omitting the $U(1)$ factors, and then the corresponding $SU(2) \times SU(2)$ theory is equivalent to the BLG theory. In general, the issue of $U(1)$ factors appears to be subtle one, and $U(1)$'s may be required for a consistent description of two M2-branes on the orbifold.

By analogy with the discussion in the previous two subsections let us now extend the 3-algebra by adding an extra generator e in a trivial way, i.e. without modifying the existing relations and with $[e, T^a; T^b] = 0$. We shall also assume that the scalar product on the extended algebra is defined so that e is perpendicular to the $\{E, \sigma^i\}$ generators and has negative norm:

$$\langle e, e \rangle = -2 \ . \tag{3.21}$$

Similarly to the $U(1) \times SU(2)$ CS example discussed above in section 3.1, let us now rescale k and rename the generators to T^+, T^-, T^i as follows

$$e = 2\varepsilon^{-1}T^- \ , \qquad E = \varepsilon T^+ + 2\varepsilon^{-1}T^- \ , \qquad \sigma^i = T^i \ , \qquad k = \varepsilon^{-1}\tilde{k} \ , \tag{3.22}$$

i.e. $T^- = \frac{1}{2}\varepsilon e$, $\quad T^+ = \varepsilon^{-1}(E - e)$. Next, let us take the limit $\varepsilon \to 0$ for fixed $T^\pm$, T^i, $\tilde{k}$. This leads to the following (totally antisymmetric) 3-algebra ($a, b = +, -, i$; $i = 1, 2, 3$)

$$[T^-, T^a, T^b] = 0 \ ,$$

$$[T^+, T^i, T^j] = f^{ij}{}_k T^k \ , \qquad f^{ij}{}_k = 2\tilde{\kappa}\epsilon^{ijk} \ , \qquad \tilde{\kappa} \equiv \frac{2\pi}{\tilde{k}} \ ,$$

$$[T^i, T^j, T^k] = -f^{ijk}T^- \ , \qquad f^{ijk} = 2f^{ij}{}_n\delta^{nk} = 4\tilde{\kappa}\epsilon^{ijk} \ . \tag{3.23}$$

Here $\tilde{\kappa}$ can be set equal to 1 by further rescaling of the generators. This is just the Lorentzian 3-algebra (2.13) associated to the gauge group $SU(2)$, with (3.21) leading to the appropriate scalar product (2.14):

$$\langle T^-, T^- \rangle = \frac{\varepsilon^2}{4}\langle e, e \rangle \to 0 \ , \qquad \langle T^+, T^+ \rangle = \varepsilon^{-2}\langle E - e, E - e \rangle = 0 \ ,$$

$$\langle T^+, T^- \rangle = \frac{1}{2}\langle E - e, e \rangle = -\frac{1}{2}\langle e, e \rangle = 1 \ . \tag{3.24}$$

In the general case of $U(N) \times U(N)$ Lie algebra, the generators T^i of $SU(N)$ satisfy

$$T^i T^j = -i\delta^{ij} E + \frac{1}{2}(if^{ij}{}_k + d^{ij}{}_k)T^k \ , \tag{3.25}$$

so that the generalization of the $N = 2$ 3-algebra relations (3.20) is

$$[E, T^i; T^j] = \kappa f^{ij}{}_k T^k \ ,$$

$$[T^i, T^j; T^k] = -N^{-1}\kappa f^{ijk}E + \kappa A^{ijk}{}_m T^m \ , \tag{3.26}$$

where the coefficients $A^{ijk}{}_m$ are determined by $f^{ij}{}_k$ and $d^{ij}{}_k$:

$$A^{ijk}{}_m T^m = N^{-1}h^{jk}T^i + \frac{1}{4}(if^{kj}{}_l + d^{kj}{}_l)(if^{il}{}_m + d^{il}{}_m)T^m - (i \leftrightarrow j) \ . \tag{3.27}$$

Unlike the $N = 2$ case (3.20) here there is an additional term in the last relation in (3.26) which is not antisymmetric in i, j, k. However, after adding an extra "ghost" generator e that commutes with all other generators and taking a similar scaling limit with

$$e = N\varepsilon^{-1}T^- \ , \qquad E = \varepsilon T^+ + N\varepsilon^{-1}T^- \ , \qquad k = \varepsilon^{-1}\tilde{k} \ , \qquad \varepsilon \to 0 \ , \tag{3.28}$$

the second term in the last line of (3.26) will vanish and we will get the antisymmetric $SU(N)$ L-BLG algebra (2.13). Note that the definition of the generators in (3.28) is in direct correspondence with the definition of fields in (3.14),(3.18):

$$Y_0 E + U e = Y_+ T^+ + Y_- T^- \ . \tag{3.29}$$

4 Concluding remarks

To conclude, the 3-algebra [21] corresponding to the ABJM theory is related to the Lorentzian antisymmetric 3-algebra corresponding to the L-BLG theory after one extends the former by a "ghost" generator and takes the scaling limit defined above. This implies that the corresponding interacting actions are also related by this scaling limit.

One may wonder if this scaling-limit relation may have some implications for the physical interpretation of the L-BLG theory, and the unitarity issue in particular. The ABJM theory extended by an abelian "ghost" supermultiplet (U^A in and the corresponding fermions) may formally be unitary assuming we define the corresponding observables (conformal operators and their correlation functions) so that they do not depend on the "ghost" fields. The observables of the L-BLG theory obtained by taking the scaling limit will then be a certain subset of all possible composite operators one could consider by starting directly with the full L-BLG action.

The relation via the scaling limit does not necessarily mean that the correlation functions of the two theories will also be directly related: while the scaling limit is smooth in the action, it need not be so at the level of the correlation functions. One may view the L-BLG theory as a certain "truncation" of the ABJM theory (similar to the case of 2d sigma models related via Penrose-type limit).

One consistency test of this scaling limit relation between the ABJM and the L-BLG theories is found by considering their known connection to 3d $\mathcal{N} = 8$ $SU(N)$ SYM theory describing N coincident D2 branes. Starting from the ABJM theory one can get the theory on D2-branes by assuming that one of the scalars Y^A develops an expectation value $\langle Y \rangle = \sqrt{k}\, v$ and then taking the limit $k \to \infty$, $v \to \infty$ with $g_{\mathrm{YM}}^2 = \frac{v^2}{k}$ kept fixed [18, 20, 30]. Since the new "ghost" field U that we introduced is completely decoupled it can be integrated out without changing this relation to 3d SYM theory. Given that the L-BLG also reduces to the D2-brane theory when one of the 8 scalars X_+^I gets an expectation value, one would naturally expect that taking the above scaling limit and then setting $\langle X_+ \rangle = \tilde{v}$ would be equivalent to the D2-brane reduction procedure of [20, 30].

Indeed, the scaling limit translates the D2-brane limit of assuming a scalar expectation value and taking it to infinity to first scaling the relevant fields to infinity and then giving one of them a finite expectation value. Comparing the resulting coupling constant g_{YM} for the two ways of getting the $\mathcal{N} = 8$ SYM theory – from the ABJM theory and from the L-BLG theory – we see that it is the same

$$g_{\mathrm{YM}} = \tilde{v} = \langle X_+ \rangle = \frac{1}{k} \langle Y \rangle = \frac{1}{k} \sqrt{k} v = \frac{v}{\sqrt{k}} = g_{\mathrm{YM}} \ . \tag{4.1}$$

Thus L-BLG theory may be interpreted as a background-independent intermediate step that can be considered when reducing from the M2-branes to the D2-branes.

The limit from the ABJM theory to D2-brane theory may be interpreted [20] as the cone $\mathbb{C}^4/\mathbb{Z}_k$ becoming locally a cylinder for large k, with the radius of the cylinder being related to the distance of the branes from the singularity, given by the expectation value of the scalar Y. The scaling limit leading to the L-BLG theory also converts the cone into a cylinder, but in this case the radius of the cylinder is a dynamical variable and only when we give it an expectation value do we recover $D2$-brane theory.

Let us note also that the scaling limit gave us the $SU(N)$ (and not $U(N)$) L-BLG from the $U(N) \times U(N)$ ABJM theory, with Y_+ related to the center-of-mass mode of the branes (cf. [31]). This interpretation is consistent with both the structure of the 3-algebras, and the shift symmetry of the action. The limit scales out half of the fluctuations and seems to move the M2 branes infinitely far from the singularity, which presumably is why we recover more supersymmetry ($\mathcal{N} = 8$ of L-BLG instead of $\mathcal{N} = 6$ of ABJM), but it also shrinks the size of the cone.

Given that the ABJM theory is dual to type IIA string theory on $AdS_4 \times \mathbb{CP}^3$ [18] (with parameters $g_s = N^{-1}(N/k)^{5/4}$, $\alpha' = \lambda^{-1/2} = \sqrt{k/N}$) one may wonder if the scaling limit leading to the L-BLG theory may mean for the dual string theory. Naively, sending k to infinity at fixed N appears to correspond to free zero-tension strings. Since the limit involves also a particular scaling of the fields, i.e. the L-BLG theory corresponds to a certain truncation of the ABJM + "ghost" theory, the possibility of a string-theory interpretation of this limit remains unclear.

Among open problems let us mention also the question of existence of other similar limits of non-antisymmetric 3-algebras in the general setting presented in [32].

Acknowledgments

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